

MathemaTIC Curriculum



Personalized Learning
Environment in Mathematics
for Every Learner



Overview

To enhance teaching and learning and to transform math education at schools, The Ministry of National Education, Children and Youth of the Grand Duchy of Luxembourg (MENJE), in collaboration with Vretta Inc. (Vretta), a leader in learning technology solutions, designed and developed the *Personalized Learning Environment* called [MathemaTIC](#) for primary and secondary school students. The purpose of *MathemaTIC* is to enhance student achievement by building a sustainable solution that is tailored to meet the specific needs of the mathematics curriculum of schools and to engage students in innovative ways to raise their level of numeracy and prepare them for success in mathematics.

To ensure the highest quality for *MathemaTIC*, the resources have been created in close collaboration and partnership with pedagogy, technology, and research experts from the following organizations:



Through the development projects, Subject Matter Experts (SME) were recruited and teams were created comprising expert professionals from SCRIPT in Luxembourg, Vretta, the University of Luxembourg, LISER, and the DEPP in France. Through collaborative efforts, the scope of the *MathemaTIC* project was established in terms of the extent of coverage of content through curriculum frameworks and the type of items required for students to successfully master challenging topic areas in each grade. The items were designed and developed in English, French, German, and Portuguese, and are currently used in Luxembourg, France, and Portugal not only as a formative assessment resource but also as a summative resource that is being used in national assessments.

MathemaTIC items are built with the belief that every learner should have access to a personalized learning environment that makes math intuitive to learn and exciting. The following lists all Domains, Sub-domains, and Learning Objectives covered in the MathemaTIC modules.

MathemaTIC Curriculum

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Grades 3 & 4

Number Sense and Numeration

Quantity Relationships

- Understanding place value from ones to thousands
- Interpreting place value charts as numbers with 4 to 7 digits
- Creating a place value chart to represent numbers with 4 to 7 digits
- Demonstrating understanding of the relationship between place values by moving dots between columns in a place value chart
- Using place value charts to compare numbers up to 4 digits (greater than, less than, equal to)
- Locating a number on a number line up to 3 or 4 digits using place value
- Decomposing numbers to facilitate addition and subtraction of up to 3-digit numbers
- Applying place value to solve numerical puzzles
- Determine what number needs to be added to a quantity to reach the next multiple of 10

Operational Sense

- Adding or subtracting 2 numbers with up to 4 digits
- Understanding subtraction of quantities through a pictorial representation
- Identifying which information presented in text is relevant to the given problem, and determining which operation should be used to determine the solution
- Following and documenting each of the steps in the 4-step problem solving process (Understand, Plan, Solve, and Check) to understand and solve problems presented in text involving up to 3 digit addition and subtraction of quantities, distance, money, and duration
- Interpreting operations represented in words (e.g. sum)
- Using 2 to 3 digit addition to solve numerical puzzles
- Decomposing a number into a product by arranging dots in an array
- Writing a number as a product of 1 digit numbers different ways
- Decomposing a number into a product of 1 digit numbers to place it in a multiplication table
- Understanding division as creating groups of equal size
- Understanding division by arranging dots in an array
- Finding divisors of a number to complete equalities involving division
- Multiplying or dividing 2 numbers with up to 2 digits
- Multiplying or dividing 2 numbers with up to 5 digits where each number has 1 significant digit (e.g. $4500 : 900 = \underline{\quad}$)
- Interpreting and solving problems presented in text involving division of 1 and 2 digit numbers
- Applying numbers and operations principles involving the 4 operations to solve complex problems involving quantity and money

- Decomposing a quantity into a sum of products using trial-and-error or other strategies



Measurement

Attributes, Units, and Measurement Sense

- Comparing amounts of money
- Following the 4-step problem solving process to solve problems presented in text involving addition and subtraction, 24-hour time and duration, money, and distance
- Setting and reading time on an analog clock
- Determining duration of time from an analog clock
- Measuring the perimeter of rectangles and right-angle polygons using a ruler or counting grid squares
- Measuring the area of right-angle polygons by counting grid squares

Measurement Relationships

- Creating a target weight by combining objects with various weights
- Converting measurements of weight between g and kg
- Creating a target capacity by combining objects with various capacities
- Converting measurements of capacity between mL and L



Geometry and Spatial Sense

Geometric Relationships, Location and Movement

- Creating complex shapes using tangrams
- Reflecting and rotating shapes using tangrams
- Navigating a complex 3D maze using map



Patterning and Algebra

Expressions and Equality

- Combining provided numbers to create equalities using up to 3 digit numbers with addition and subtraction
- Combining provided numbers to create equalities using 1 to 2 digit numbers with multiplication and division
- Combining provided numbers to create equalities using up to 3 digit numbers with all 4 operations
- Determining the missing value to add or subtract from one side to achieve equality
- Complete number sentences involving addition or subtraction of 2 numbers (up to 4 digits) where any of the 3 numbers could be missing (e.g. $__ + 75 = 77$)
- Complete number sentences involving multiplication or division of 2 numbers (up to 2 digits) where any of the 3 numbers could be missing (e.g. $__ : 2 = 9$)
- Complete number sentences involving multiplication and division of 2 numbers up to 5 digits where each number has 1 significant digit (e.g. $2500 : __ = 500$)

Grades 5 & 6



Number Sense and Numeration

Quantity Relationships

- Understanding the parts of a fraction - numerator and denominator
- Locating and visualizing fractions on the number line
- Determining a fraction given its location on the number line
- Creating a visual representation of a fraction by colouring elements of a set
- Representing and interpreting a fraction as a part of figure's area
- Finding common multiples of 2 numbers
- Determining the lowest common multiple of 2 numbers
- Creating and identifying equivalent fractions
- Determining missing numerator or denominator to make equivalent fractions
- Comparing fractions by using equivalent fractions to place fractions on the number line
- Ordering fractions from smallest to largest and from largest to smallest
- Identifying a fraction that lies between two fractions
- Determining the area that represents a given percentage of a total area
- Identifying odd and even numbers
- Rounding up to 7-digit numbers to the nearest tens, hundreds, thousands or ten thousands
- Applying place value to solve numerical puzzles

Operational Sense

- Using a variety of strategies (such as decomposing numbers, applying commutativity or associativity) to complete addition and subtraction exercises up to 3 digits
- Estimating results of addition/subtraction expressions with up to 4 digit numbers
- Using multiplication to count objects arranged into an array of arrays
- Decomposing a number into two factors in different ways
- Applying divisibility criteria for 2, 5, 10, 4, and 3 to determine if numbers up to 6 digits are divisible by given numbers
- Using a variety of strategies (such as decomposing numbers, applying commutativity, associativity, or distributivity) to complete multiplication exercises
- Identifying the relationship between similar products and quotients, and using this to facilitate computation
- Adding and subtracting fractions with like and unlike denominators
- Multiplying fractions
- Applying multiplication of fractions to solve numerical puzzles
- Solving problems involving division with remainder

Proportional Relationships

- Using fractions to describe a proportional relationship involving manipulatives (gears)
- Recreating a geometric puzzle by proportionally increasing or decreasing the size of each shape
- Dividing an area into 100 equal squares to represent percentages

- Writing a fraction with denominator of 100 or a multiple of 100 as a percentage
- Determining the area that is represented by a percentage of the total area



Measurement

Attributes, Units, and Measurement Sense

- Understanding perimeter as the total distance around a figure, and not including any distances in the interior of a figure
- Calculating the perimeter of rectangles, parallelograms, triangles, right-angled polygons, and irregular polygons using rulers or grid squares
- Understanding area as the total surface occupied by a figure
- Discovering the formula for the area of a rectangle by filling the shape with grid squares, and linking this with multiplication and the measurements of length and width
- Calculating the area of rectangles, squares, parallelograms, triangles, and right-angled polygons using formulas and rulers, or counting grid squares
- Estimating the area and perimeter of an irregular figure

Measurement Relationships

- Constructing rectangles with a given perimeter or area
- Investigating how different formulas for the perimeter of rectangles and squares are equivalent
- Discovering the formula for the area of a parallelogram by cutting different examples into pieces and rearranging them to form a rectangle
- Discovering the formula for the area of a triangle by duplicating and rotating the shape to form a parallelogram
- Comparing the areas of rectangles, triangles, and parallelograms
- Comparing the areas of figures constructed from tangram pieces
- Creating shapes of equal area on a grid
- Converting metric units of length, area, and volume
- Ordering measurements of length, area, and volume
- Using knowledge of perimeter to solve problems involving the shortest distance between 2 points on a figure along its perimeter



Geometry and Spatial Sense

Geometric Properties

- Sorting 3D solids into curved surfaces, pyramids, prisms, and others
- Counting the faces, edges, and vertices on 3D solids
- Identifying different types of angles (acute, obtuse, right, and straight)
- Using protractors to measure angles
- Calculating the volume of a 3D solid constructed out of cube blocks by counting blocks
- Calculating the surface area of a 3D solid constructed out of cube blocks by counting the visible block faces

Geometric Relationships

- Creating 2D nets for 3D solids
- Matching 2D nets to 3D solids
- Exploring Euler's characteristic ($F+V-E=2$) with a variety of solids

- Adding cube blocks to 3D solids constructed out of cube blocks to form cubes or rectangular prisms
- Creating angles by adding and subtracting other angles
- Visualizing different 2D views of 3D solids

Location and Movement

- Identifying x- and y-coordinates of an ordered pair, and linking them to locations on the Cartesian plane
- Plotting points on the Cartesian plane given coordinates
- Creating familiar geometric shapes by plotting points on a Cartesian plane
- Translating 2D shapes, with and without a grid
- Performing axial reflections, with and without a grid
- Performing radial reflections, with and without a grid
- Performing rotations without a grid
- Choosing or placing the point that is symmetric to a given point across a vertical, horizontal, or diagonal axis
- Identifying and applying rotations and reflections needed to continue a pattern made from tiles
- Applying knowledge of parallel and perpendicular lines to identify locations on a map



Patterning and Algebra

Patterns and Relationships

- Understanding a numerical expression as a sequence of operations
- Simplifying a sequence of operations to an equivalent sequence
- Reversing a sequence of operations to find the initial number, given a result
- Finding the missing operation in a sequence of operations to reach the desired result, given an initial number
- Following a computation program involving addition, multiplication, and division
- Understanding a geometric linear growing pattern and determining characteristics of different stages of the pattern by building the shape and extrapolating the pattern
- Generating a strategy for finding higher terms in a geometric linear growing pattern
- Identifying the next term in a numerical sequence (using one of the 4 operations)

Variables, Expressions, and Equality

- Exploring and using the commutative property of addition and multiplication to facilitate computations
- Exploring and using the associative property of addition and multiplication to facilitate computations
- Exploring and using the distributive property of multiplication over addition and subtraction to facilitate computations
- Finding the value of a complex algebraic expression given values for all variables, and determining the value for one variable needed to obtain a desired result
- Follow a schematic to identify key parts in a sequence of numbers and interpret the results
- Solving algebraic equations with 1 or 2 variables through trial-and-error



Data Management and Probability

Collection and Organization of Data

- Creating a bar graph, line graph, or pie chart to represent data
- Generating data and creating a line graph to display the data
- Visualizing proportions in a pie chart, adjusting the number of quantities to create certain proportions
- Identifying key information from text
- Inputting data correctly into tables with given row and column labels

Data Relationships

- Generating points on a scatterplot graph to find where two linear equations intersect
- Interpreting data from a bar graph, line graph, or pie chart
- Generating data and entering it into a table to draw conclusions on the data

Grades 7 & 8



Number Sense and Numeration

Quantity Relationships

- Generating lists of multiples, identifying common multiples
- Interpreting and representing fraction as a part of a whole
- Representing a fraction visually as a part of a shape's area
- Determining the fraction represented by an area
- Identifying the piece of a shape that covers a specified fraction of its area
- Placing and interpreting fractions on a number line by dividing the unit into an appropriate number of parts
- Understanding, identifying, and creating equivalent fractions by amplifying and simplifying
- Creating an equivalent fraction with a specified numerator or denominator
- Interpreting several representations of fractional values (including written fractions, different visual representations, and decimal form) and determining which are equivalent
- Comparing fractions using a variety of strategies (including creating equivalent fractions with a common numerator or denominator, placing the fractions on a number line, or simplifying before comparing)
- Ordering fractions from smallest to largest and from largest to smallest
- Converting decimals to fractions using place value
- Converting fractions to decimals through different strategies (creating an equivalent fraction with a power of 10 as the denominator, or dividing the numerator by the denominator)
- Understanding negative numbers as numbers that are below 0 (using temperature and the number line as a reference)
- Sorting integers into negative and positive numbers
- Placing integers on a number line

- Understanding the concept of absolute value and opposite numbers
- Comparing integers using a number line
- Ordering positive and negative numbers (including decimals) from smallest to largest and from largest to smallest
- Applying knowledge of sums, squares, and comparing positive whole numbers to solve numerical puzzles

Operational Sense

- Using operation terminology (e.g. sum, product, difference, quotient) to create and identify specified expressions
- Creating expressions to result in specified sums, differences, products, and quotients
- Using order of operations to identify which operations to perform first in an expression
- Differentiating between expressions that are sums or products by applying order of operations
- Combining available numbers and operations to create an expression with a target result
- Exploring the distributive property of multiplication over addition through an area model
- Applying divisibility criteria for 2, 3, 4, 5, 6, 9, and 10 to determine if numbers up to 6 digits are divisible by a specified number
- Creating numbers that are divisible by specified number(s)
- Identifying all divisors of a number
- Identifying common divisors of 2 numbers
- Adding and subtracting fractions with common and different denominators
- Multiplying 2 or 3 fractions
- Identifying patterns in multiplying fractions with numerator or denominator 1 in order to facilitate computations
- Dividing a whole number by a fraction
- Dividing 2 fractions
- Simplifying a fraction with a fraction in the numerator or denominator
- Applying order of operations to a fraction with operations in the numerator or denominator
- Converting a numerical expression with integers into simplified form (e.g. $(-3) - (+4) = -3 - 4$)
- Adding and subtracting integers with up to 2 digits and up to 3 terms
- Representing addition and subtraction of integers on a number line
- Decomposing integers to facilitate addition and subtraction
- Combining available integers to create a sum with the desired result
- Explore integer multiplication through repeated addition
- Explore integer division through its connection to integer multiplication
- Multiply and divide 1-2 digit integers
- Computing a percentage of a quantity through various methods (including converting the percentage to a decimal, converting the percentage into a fraction, and calculating 1% of the quantity and then scaling up)
- Calculating the percent of a quantity with a certain characteristic (e.g. What percentage of letters in "percentage" are vowels?)

- Calculating percentages and percentages of a quantity from real-world situations
- Applying knowledge of sums to solve numerical puzzles

Proportional Relationships

- Using fractions to obtain a specified part of the total volume
- Understanding percentages as representing a fraction with denominator 100
- Representing fractions with denominators other than 100 as percentages
- Using percentage as a way of fairly comparing results of data with different sample sizes
- Discovering that percentages are proportional to the quantities they represent, and using linearity to solve for unknown percentages
- Estimating the percentage of a bar that has been filled
- Explore and define the proportionality coefficient
- Calculating the proportionality coefficient in a proportional relationship
- Using the proportionality coefficient to determine unknowns in a proportional relationship
- Using the proportionality coefficient to prove that 2 quantities are not proportional
- Discover the additive property of proportional relationships
- Using additivity to determine unknowns in a proportional relationship
- Using the additive property of proportional relationships to prove that 2 quantities are not proportional
- Discover the linearity property of proportional relationships
- Using linearity to determine unknowns in a proportional relationship
- Explore the structure of the graph comparing 2 proportional quantities
- Determining from graphs which 2 quantities may be proportional
- Extrapolating and interpolating a graph comparing 2 proportional quantities to determine unknowns in a proportional relationship
- Explore why properties of proportionality cannot prove that 2 quantities are proportional based on a finite data set
- Filling in a proportionality table using various methods (coefficient of proportionality, linearity, or additivity)
- Explore the concept of speed as a measure of distance covered over a set period of time
- Explore how to convert speeds between km/h and m/s by converting the measures of distance and time separately, and using this to develop a formula to facilitate this conversion
- Identifying what units and quantities are used in a word problem (e.g. time, distance, meters, seconds)
- Using speed and elapsed time to calculate distance
- Measuring path distances on a map using a scale
- Solving problems in real-world situations using map scales and speed
- Solving problems in real-world situations using a unit rate
- Programming a spreadsheet to represent proportional and non-proportional relationships
- Using a programmed spreadsheet to solve for unknowns by entering the known values
- Identify proportional quantities in a real-world situation, and use properties of proportional quantities to determine unknowns in that situation (e.g. coefficient of proportionality, linearity, or additivity)

- Using knowledge of proportional relationships to solve complex problems incorporating proportional and non-proportional quantities
- Finding proportional relationships in everyday situations and presenting these ideas in a video
- Graphing a geometric relationship to determine if the measurements are proportional or not



Measurement

Attributes, Units, and Measurement Sense

- Discovering the formula for the perimeter of a circle through investigation and measurement
- Calculating the perimeter of shapes (including rectangles, right-polygons irregular polygons, and circles)
- Calculating the area of familiar shapes (including rectangles, parallelograms, triangles, trapezoids, and circles)
- Representing an algebraic expression as an area with variables in the dimensions
- Discovering the formula for the volume of right prisms and cylinders
- Calculating the volume of right prisms and cylinders
- Calculating the surface area and lateral surface area of right prisms and cylinders
- Calculating the volume of pyramids and cones

Measurement Relationships

- Discovering the formula for the area of a parallelogram by cutting different examples into pieces and rearranging them to form a rectangle
- Discovering the formula for the area of triangles and trapezoids by duplicating the shape and rotating it to form a parallelogram
- Discovering the formula for the area of a circle through investigation resulting in a rectangle with the same area
- Creating shapes (including rectangles, parallelograms, trapezoids, and circles) with a specified perimeter
- Creating shapes (including rectangles, parallelograms, trapezoids, and circles) with a specified area
- Calculating the perimeter and area of complex composite shapes by decomposing them into familiar shapes
- Discovering the formula for the volume of pyramids and cones by investigating the relationship with the volume of right prisms
- Calculating the volume of composite solids by decomposing them into familiar solids
- Comparing measurements of length, area, volume, and capacity
- Converting metric units of length, area, volume, and capacity



Geometry and Spatial Sense

Geometric Properties

- Identifying acute, obtuse, right, and reflex angles
- Distinguishing between positive and negative angles

- Understanding that a perpendicular bisector is perpendicular to a segment, and divides it into 2 segments of equal length
- Using perpendicular bisectors to identify points that are closer to either endpoint of the segment
- Constructing a perpendicular bisector with straightedge and compass
- Identifying acute, obtuse, right, isosceles and equilateral triangles
- Discover different types of quadrilaterals and investigate some of their properties of different quadrilaterals (e.g. number of parallel sides, equal angles, sides of equal length, and properties of diagonals)
- Determining the properties of different quadrilaterals (e.g. number of parallel sides, equal angles, sides of equal length, and properties of diagonals)
- Constructing quadrilaterals using diagonal properties

Geometric Relationships

- Constructing triangles with specified side lengths and angles
- Investigating the requirements to specify a unique triangle (e.g. 3 side lengths)
- Explore the triangle inequality and use it to identify lengths that cannot be used to construct a triangle
- Using a graph to compare geometric properties of shapes (such as side length, area or perimeter), and using the graph to determine unknown values
- Calculating the amplitude of adjacent angles
- Identifying vertical, complementary, interior, alternate interior angles, and using these angle types to find amplitudes of angles
- Discover through investigation that the sum of the interior angles of a triangle is 180 degrees, and use this to find the amplitudes of angles in a triangle
- Determining the amplitudes of unknown angles using the properties of vertical, complementary, the interior angles of a triangle, and the properties of equilateral and isosceles triangles
- Determining which quadrilaterals are special cases of others based on their properties

Location and Movement

- Exploring transformations of the plane (including rotations, axial reflections, curved reflections, translations, rotations, and resizing)
- Examining properties preserved by some transformations, such as amplitudes of angles
- Understanding the definition of an isometry
- Exploring axial and central reflection, and identifying reflected shapes
- Constructing axial and central reflections using straightedge and compass
- Examining properties preserved by axial and central reflections (including distances, point alignment, amplitude of angles, and circle properties)
- Identifying axial and central symmetries in a variety of shapes (both geometric and from real-life)
- Creating geometric shapes using axial and central reflections of given figures
- Identifying the axial reflection needed to transform a point onto a specified image
- Exploring, identifying, and performing rotations

- Examining properties preserved by rotations (including distances, point alignment, amplitude of angles, and circle properties)
- Recreating a pattern with rotational symmetry by identifying the number of rotational symmetries and placing geometric shapes
- Exploring, identifying, and performing translations
- Examining properties preserved by translations (including distances, point alignment, amplitude of angles, and circle properties)
- Using translations to control an avatar's movement on a 2D plane
- Recreating a pattern using a combination of isometries
- Creating the image of shapes under specified isometries
- Completing sequences of isometries to navigate a room with obstacles
- Understanding notation of isometries to identify the image of specified points



Patterning and Algebra

Patterns and Relationships

- Representing an algebraic expression in a variety of ways (including with a balance model, pictorially with blank boxes in place of variables, and in words)
- Exploring geometric linear growing patterns, create a table of values to represent the first few terms
- Creating an algebraic formula to represent a geometric linear growing pattern and determine later terms in the sequence
- Exploring Fibonacci patterns, using a spreadsheet to represent the pattern and determine later terms in the sequence
- Identifying a pattern in a sequence of numbers and translating it into an algebraic expression

Variables, Expressions, and Equations

- Understanding the role of variables to represent unknowns or values that can vary with real-world examples
- Identifying variables, factors, terms, and coefficients in an algebraic expression
- Creating and identifying equivalent algebraic expressions
- Proving that two algebraic expressions are not equivalent by finding a value for the variable that serves as a counter-example
- Simplifying algebraic expressions (by removing unneeded multiplication signs and brackets, reordering variables and coefficients, using exponents to represent repeated multiplication, removing coefficients of 1 and terms of 0, applying distributivity of multiplication over addition and subtraction)
- Understanding like terms through use of manipulatives
- Simplifying algebraic expressions by combining like terms
- Determining the value of an algebraic expression given values for all of its variables
- Creating algebraic expressions to represent geometric quantities for shapes with variable side lengths
- Generating data based on a real-life situation, creating an algebraic expression to model the data, and using the algebraic model to answer questions about the real-life situation

- Creating a sequence of operations to describe an algebraic expression
- Creating an algebraic expression to represent an expression in words, a real-world situation, or a set of manipulatives
- Developing and presenting an algebraic formula to represent a real-world situation through a video
- Expanding algebraic expressions by applying the distributivity property of multiplication over addition and subtraction
- Identifying common factors (numerical, variable, or both) in an algebraic expression to factor the expression
- Programming formula into a spreadsheet to help solve algebraic equations by trial-and-error, and testing the spreadsheet formula to ensure it has been entered correctly
- Solving a linear equation in 1 variable (potentially including brackets, or with variables on both sides of the equality) through trial-and-error using the balance method or a spreadsheet, and systematic algebraic methods
- Finding a solution to a linear equation in 2 variable through trial-and-error using the balance method, and justifying that there are infinitely many solutions
- Exploring linear equations in 1 variable that have no solutions, or infinitely many solutions
- Creating and solving algebraic equations in 1 variable to solve geometric problems involving the area or perimeter of shapes with variable side lengths



Data Management and Probability

Collection and Organization of Data

- Plotting points from a table on a Cartesian grid
- Gathering survey data, entering results of survey data in a table
- Extracting data from text and entering it into a table with provided row and column labels
- Using a table to organize values of an algebraic expression for specified values of each variable

Data Relationships

- Creating scatter plot graphs from a table of values to test quantities for proportionality
- Creating a bar graph from a table of data with a title, axis titles, and appropriate scale
- Creating a pie chart from a table of data
- Interpreting a pie chart to answer questions on the data
- Reading a line graph to answer questions on the data
- Computing range, frequency, sample size, mode, average, median, and relative frequency of a data set
- Computing relative frequency as a fraction and a percentage
- Comparing frequency to relative frequency
- Computing sample size, relative frequency, or frequency, given values for the other 2
- Interpreting range, average and median within a context
- Creating data sets with specified averages, medians, or ranges